A Dispersion Mismatch Criterion for the Main Injector to Tevatron Transfer Line

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Care must be taken in the design of beamlines to match the lattice parameters between the rings. The failure to do so can result in unwanted emittance growth. The most difficult of these parameters to match is the dispersion function, however this is the most critical in that small mismatches can lead to significant emittance growth. In the beam lines between the Main Injector and the Tevatron it may not be possible to completely match the vertical dispersion function. Although both rings are planar, and thus have no vertical dispersion, the beamline itself will generate the dispersion by virtue of the vertically bending dipoles in the line. The question is that of how much mismatch can be allowed.

The starting point for this analysis is the standard phase space dilution equation. [1]

$$\frac{\varepsilon}{\varepsilon_0} = 1 + \frac{1}{2}\left(\frac{\Delta D_{eq}\left(\frac{\sigma_p}{p}\right)}{\sigma_0}\right)^2$$

This equation expresses the ratio of the resulting emittance due to the mismatch to the undiluted emittance. This ratio is a function of the rms of the momentum spread, the rms of the beam distribution, and a quantity called the equivalent dispersion error which is defined as:

$$\Delta D_{eq} \equiv \sqrt{\Delta D^2 + (\beta \Delta D' + \alpha \Delta D)^2}$$

Where the ΔD and ΔD' are the differences between the required dispersion and the actual dispersion functions. In the case of vertical dispersion matching the required dispersion is zero, so these become the values of the dispersion function in the beam line after the final bend. For our purposes we rewrite this equation as

$$\varepsilon = \varepsilon_0 + \frac{\varepsilon_0}{2}\left(\frac{\Delta D_{eq}\left(\frac{\sigma_p}{p}\right)}{\sigma_0}\right)^2$$

or

$$\Delta\varepsilon = \frac{\varepsilon_0}{2}\left(\frac{\Delta D_{eq}\left(\frac{\sigma_p}{p}\right)}{\sigma_0}\right)^2$$

Next, expressing the original emittance as a function of the rms beam distribution yields:

$$\Delta\varepsilon = \frac{6\pi\sigma_0^2(\beta\gamma)}{2\beta}\left(\frac{\Delta D_{eq}\left(\frac{\sigma_p}{p}\right)}{\sigma_0}\right)$$

which can be written as:

$$\frac{\Delta\varepsilon}{\pi} = 3(\beta\gamma)\frac{\Delta D_{eq}^2}{\beta}\left(\frac{\sigma_p}{p}\right)^2$$

The quantity $\frac{\Delta D_{eq}^2}{\beta}$ is an invarient as long as no dipoles are encountered. Therefore the amount of emittance growth is determined from the energy, momentum spread, and an invarient quantity that is easily derived from the point of the the last vertical dipole in the transport line.

Of the various running modes of the Main Injector, the one in which the emittance growth is most critical is the one in which particles are destined for collision in the Tevatron. However this mode may present the largest momentum spread due to the coalescing process that occurs prior to transfer. The largest rf bucket anticipated is 4eV - sec. We assume that the bucket is filled and calculate the resulting momentum spread to be 325 MeV p-p. At 150 GeV this is a Δp/p of .218%, and converting this to an rms yields .0487%.

The fly at 150 GeV is 160 so that the above equation is:

$$\frac{\Delta\varepsilon}{\pi} = 480 \times .487^2 \times \frac{\Delta D_{eq}^2}{\beta}$$

Where the momentum spread has been written in per mil so that the result can be taken as mm - mrad.

This then sets the criterion for allowable dispersion mismatch. If 1 π mm - mrad emit tance growth is allowable, then the quantity 9 must be kept less than .0088.

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[1]M. Syphers, "Injection Mismatch and Phase Space Dilution", p. 32

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